KINEMATICS OF A CONTINUOUSLY DISLOCATED MEDIUM

(KINEMATIKA KONTINUAL'NO – DISLOKATSIONNOI SREDY)

PMM Vol. 30, No. 5, 1966, pp. 950-955

V.A. BABKIN
(Moscow)

(*Received November 29, 1965*)

The theory of deformation of a medium with a continuous distribution of dislocations is studied. Such a medium will be called continuously dislocated, in contradistinction to a discretely dislocated medium in which the number of dislocations is finite, albeit very large.

1. Deformation. The motion of a point in a solid medium with Lagrangian coordinates ξ^i $(i = 1, 2, 3)$ is examined in three-dimensional Euclidean space, in which we introduce a Cartesian coordinate system x^i $(i = 1, 2, 3)$. All kinematic properties will be referred to the x^i-axes.

The present work deals with the following fundamental hypothesis: At a fixed time t, a small relative displacement of the points with Lagrangian coordinates ξ^i and $\xi^i + d\xi^i$ is the total differential with respect to ξ^i and certain, as yet undefined, physical parameters $\chi^s(\xi^i, t)$ $(s = 1, ..., S)$.

$$d\mathbf{u} = \frac{\partial \mathbf{u}}{\partial \xi^i} d\xi^i + \frac{\partial \mathbf{u}}{\partial \chi^s} d\chi^s \tag{1.1}$$

where $\mathbf{u} = u^i(\xi^j, \chi^s(\xi^j, t), t)\ \ni i$ is the displacement of a point in Cartesian coordinates.

Hereinafter, summation over paired indices is understood.

By definition, a relative displacement with $\chi^s = \text{const}$ $(s = 1, 2, ..., S)$

$$d\mathbf{u}_1 = \frac{\partial \mathbf{u}}{\partial \xi^i}\bigg|_{\chi^s = \text{const}} d\xi^i \tag{1.2}$$

is an elastic relative displacement, while a relative displacement

$$d\mathbf{u}_2 = \frac{\partial \mathbf{u}}{\partial \chi^s} d\chi^s \tag{1.3}$$

is a parametric (inelastic) relative displacement, because it is related to a change in the parameters χ^s.

The physical reason for introducing the parameters χ^s is, that their introduction allows for the division of small relative displacements into two parts, (1.2) and (1.3). Thus, the kinematic description becomes more detailed: The vector $\mathbf{u}(\xi^i, \chi^s, t)$ indicates not only the general displacement of the point ξ^i, but also the physical processes in the medium, described by the parameters χ^s, which accompany the displacement.

The parameters χ^s which depend on the coordinates ξ^i and t, by the very reason for their introduction, must be closely related to the relative displacement, characterizing it at every point and at every moment. They may be scalars, vectors or tensors, but they must generally be found by experiment and from physical considerations.

The mathematical reason for introducing χ^s is, that (1.1) permits the description of relative displacements in a continuously dislocated medium.

Let us rewrite (1.1) in the form

$$d\mathbf{u} = \left(\frac{\partial \mathbf{u}}{\partial \xi^i} + \frac{\partial \mathbf{u}}{\partial \chi^s}\frac{\partial \chi^s}{\partial \xi^i}\right) d\xi^i = \frac{D\mathbf{u}}{D\xi^i} d\xi^i \tag{1.4}$$

Since $d\mathbf{u}$ is the total differential with respect to ξ^i, we have

$$\frac{D^2\mathbf{u}}{D\xi^i D\xi^j} = \frac{D^2\mathbf{u}}{D\xi^j D\xi^i} \tag{1.5}$$

and since $d\mathbf{u}$ is also the total differential with respect to ξ^i and χ^s (see the hypotheses above), we have

$$\frac{\partial^2\mathbf{u}}{\partial\xi^i \partial\xi^j} = \frac{\partial^2\mathbf{u}}{\partial\xi^j \partial\xi^i} \tag{1.6}$$

But it can be easily verified that

$$\frac{D}{D\xi^j}\frac{\partial \mathbf{u}}{\partial \xi^i} \neq \frac{D}{D\xi^i}\frac{\partial \mathbf{u}}{\partial \xi^j} \tag{1.7}$$

Hence, the integral over any closed contour C given by

$$\oint_C \frac{\partial \mathbf{u}}{\partial \xi^i} d\xi^i = \mathbf{b} \tag{1.8}$$

yields, by definition [1 and 2], a Burgers vector which, in view of (1.7), is non-zero. This implies that the medium under consideration has continuously distributed dislocations.

If the relative displacement $d\mathbf{u}$ is given by (1.2) and (1.3), then, clearly the medium contains no dislocations.

The Burgers vector may also be given by the integral

$$\oint_C \frac{\partial \mathbf{u}}{\partial \chi^s}\frac{\partial \chi^s}{\partial \xi^i} d\xi^i = -\mathbf{b} \tag{1.9}$$

In the above formulas, the symbol $D\mathbf{u}/D\xi^i$ denotes the total partial derivative with respect to ξ^i, which takes into account the dependence of $\mathbf{u}$ on $\chi^s(\xi^i)$. The quantity $\partial\mathbf{u}/\partial\xi^i$ is the partial derivative with respect to ξ^i considering χ^s constant. This is the elastic partial derivative.

The components of the final strain tensor are given by [3]

$$2\varepsilon_{ij} = g\hat{}_{ij} - g^\circ_{ij} = \partial\hat{}_i\partial\hat{}_j - \partial^\circ_i\partial^\circ_j \tag{1.10}$$

where $\partial\hat{}_i$ form the basis in moving Lagrangian coordinates while ∂°_i form the basis in fixed coordinates.

Utilizing known methods of expressing the strain tensor in terms of displacements [3] and Formula (1.4), we readily obtain, in the Cartesian coordinate system x^i,

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x^j}+\frac{\partial u_j}{\partial x^i}-\frac{\partial u_\alpha}{\partial x^i}\frac{\partial u^\alpha}{\partial x^j}+\frac{\partial u_i}{\partial \chi^s}\frac{\partial \chi^s}{\partial x^j}+\right.$$

$$\left.+\frac{\partial u_j}{\partial \chi^s}\frac{\partial \chi^s}{\partial x^i}-\frac{\partial u_\alpha}{\partial x^i}\frac{\partial u^\alpha}{\partial \chi^s}\frac{\partial \chi^s}{\partial x^j}-\frac{\partial u_\alpha}{\partial x^j}\frac{\partial u^\alpha}{\partial \chi^s}\frac{\partial \chi^s}{\partial x^i}-\frac{\partial u_\alpha}{\partial \chi^s}\frac{\partial u^\alpha}{\partial \chi^t}\frac{\partial \chi^s}{\partial x^i}\frac{\partial \chi^t}{\partial x^j}\right) \qquad (1.11)$$

For infinitesimal displacements,

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x^j}+\frac{\partial u_j}{\partial x^i}+\frac{\partial u_i}{\partial \chi^s}\frac{\partial \chi^s}{\partial x^j}+\frac{\partial u_j}{\partial \chi^s}\frac{\partial \chi^s}{\partial x^i}\right) \qquad (1.12)$$

The strains in (1.12) may be divided into elastic $\varepsilon^{(e)}{}_{ij}$ and inelastic, or parametric $\varepsilon^{(p)}{}_{ij}$

$$\varepsilon_{ij}=\varepsilon^{(e)}_{ij}+\varepsilon^{(p)}_{ij} \qquad (1.13)$$

where

$$\varepsilon^{(e)}_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x^j}+\frac{\partial u_j}{\partial x^i}\right), \qquad \varepsilon^{(p)}_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial \chi^s}\frac{\partial \chi^s}{\partial x^j}+\frac{\partial u_j}{\partial \chi^s}\frac{\partial \chi^s}{\partial x^i}\right) \qquad (1.14)$$

Similarly, the tensor of infinitesimal rotations may be represented as the sum of two components

$$\eta^{(e)}_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x^j}-\frac{\partial u_j}{\partial x^i}\right), \qquad \eta^{(p)}_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial \chi^s}\frac{\partial \chi^s}{\partial x^j}-\frac{\partial u_j}{\partial \chi^s}\frac{\partial \chi^s}{\partial x^i}\right) \qquad (1.15)$$

However, neither the tensor of finite strain $\varepsilon_{ij}\partial^i\partial^j$ nor the tensor of finite rotation $\eta_{ij}\partial^i\partial^j$ may be split into elastic and parametric parts, as may be seen, for example, from (1.11).

For the medium under investigation

$$\mathbf{u}=\mathbf{r}-\mathbf{r}_0=\mathbf{u}\,(\xi^i,\,\chi^s\,(\xi^i,\,t),\,t) \qquad (1.16)$$

The velocity of the point ξ^i is given by

$$\mathbf{v}=\left.\frac{\partial \mathbf{u}}{\partial t}\right|_{\chi^s,\,\xi^i=\mathrm{const}}+\left.\frac{\partial \mathbf{u}}{\partial \chi^s}\frac{\partial \chi^s}{\partial t}\right|_{\xi^i=\mathrm{const}}=\mathbf{v}^{(e)}+\mathbf{v}^{(p)} \qquad (1.17)$$

The components of the strain rate tensor are given by [3]

$$\oint_C \frac{\partial \mathbf{u}}{\partial \xi^i}\,d\xi^i=\mathbf{b} \qquad (1.18)$$

Taking into account (1.14), (1.17) and (1.18), we obtain the components of the strain rate tensor from the velocity in case when $\partial u_i/\partial x^j$ does not depend explicitly on χ^s and $\partial u_i/\partial \chi^s$ does not depend explicitly on t:

$$e^{(e)}_{ij}=\frac{1}{2}\left(\frac{\partial v^{(e)}{}_i}{\partial x^j}+\frac{\partial v^{(e)}{}_j}{\partial x^i}\right), \qquad e^{(p)}_{ij}=\frac{1}{2}\left(\frac{\partial v^{(p)}{}_i}{\partial \chi^s}\frac{\partial \chi^s}{\partial x^j}+\frac{\partial v^{(p)}{}_j}{\partial \chi^s}\frac{\partial \chi^s}{\partial x^i}\right) \qquad (1.19)$$

2. Geometric treatment of strain theory. Let us examine the strains of a continuously dislocated medium, and introduce a moving Lagrangian coordinate system with the basis

$$\vartheta^\wedge_i = \frac{D\mathbf{r}}{D\xi^i} \tag{2.1}$$

If the medium under study is considered as some space whose metric is

$$g^\wedge_{ij} = \vartheta^\wedge_i \vartheta^\wedge_j = g^\circ_{ij} + 2\varepsilon^\wedge_{ij} \tag{2.2}$$

where $g^\circ_{ij} = \vartheta^\circ_i \vartheta^\circ_j$ and ϑ°_i form a basis in the initial state in Euclidean space, then it turns out that the above space is Euclidean.

In fact, it is easily shown by direct computation that

$$R_{ikl}{}^m = \frac{\partial \Gamma_{kl}{}^m}{\partial x^i} - \frac{\partial \Gamma_{il}{}^m}{\partial x^k} + \Gamma_{ip}{}^m \Gamma_{kl}{}^p - \Gamma_{kp}{}^m \Gamma_{il}{}^p \equiv 0 \tag{2.3}$$

$$S_{ij}{}^k = \frac{1}{2}(\Gamma_{ij}{}^k - \Gamma_{ji}{}^k) \equiv 0 \tag{2.4}$$

Here $R_{ikl}{}^m$ are the components of the curvature tensor, $S_{ij}{}^k$ are the components of the torsion tensor and $\Gamma_{ij}{}^k$ are the coefficients of connection written in Cartesian coordinates. The coefficients of connection are defined by

$$\frac{D\vartheta^\wedge_i}{D\xi^j} = \Gamma^\wedge_{ij}{}^k \vartheta^\wedge_k = \Gamma_{ij}{}^k \vartheta_k \tag{2.5}$$

This result is entirely natural, since the entire strain of the body is the result of transference of each point in the body from one Euclidean space into another Euclidean space, i.e. the body remains a part of Euclidean space.

But the continuously dislocated medium under study may be considered a space in which the metric tensor $g^{(e)}_{ij}\vartheta^i\vartheta^j$ is defined by

$$g^{(e)\wedge}_{ij} = \vartheta^{(e)}_i \vartheta^{(e)}_j, \qquad g^{(e)\wedge}_{ij}\, \vartheta^{(e)i} \vartheta^{(e)j} = g^{(e)}_{ij}\, \vartheta^i \vartheta^j \tag{2.6}$$

$$\vartheta^{(e)}_i = \left.\frac{\partial \mathbf{r}}{\partial \xi^i}\right|_{\chi^s = \mathrm{const}} \tag{2.7}$$

$\vartheta^{(e)}_i$ $(i = 1, 2, 3)$ will be called the elastic basis.

From (1.10), (2.6) and (1.14), we find that, for small strains, the metric tensor $g^{(e)}_{ij}\vartheta^i\vartheta^j$ is related to the tensor of elastic strains by the formulas

$$g^{(e)\wedge}_{ij} - g^\circ_{ij} = 2\varepsilon^{(e)\wedge}_{ij} \tag{2.8}$$

Inasmuch as motion of the medium will result in the dependence of $\vartheta^{(e)}_i$ on the coordinates ξ^i, explicitly as well as through the parameters χ^s, the coefficients of connection are given by

$$\frac{D\vartheta^{(e)}_i}{D\xi^j} = \Gamma^{(e)\wedge}_{ij}{}^k \vartheta^{(e)}_k = \Gamma^{(e)}_{ij}{}^k \vartheta_k \tag{2.9}$$

Direct computation in Cartesian coordinates yields

$$2S^{(e)}_{ij}{}^k \vartheta_k = (\Gamma^{(e)}_{ij}{}^k - \Gamma^{(e)}_{ji}{}^k)\,\vartheta_k = \frac{D}{Dx^j}\frac{\partial \mathbf{u}}{\partial x^i} - \frac{D}{Dx^i}\frac{\partial \mathbf{u}}{\partial x^j} \tag{2.10}$$

$$\frac{1}{2}\left(\frac{Dg^{(e)}_{is}}{Dx^j} + \frac{Dg^{(e)}_{js}}{Dx^i} - \frac{Dg^{(e)}_{ij}}{Dx^s}\right) -$$

$$- S^{(e)}_{js}{}^k g^{(e)}_{ki} - S^{(e)}_{is}{}^k g^{(e)}_{kj} + S^{(e)}_{ij}{}^k g^{(e)}_{ks} = \Gamma^{(e)}_{ij}{}^k g^{(e)}_{ks} \tag{2.11}$$

Formulas (2.10) and (2.11) show that a continuously dislocated medium may, from a geometric point of view, be considered as a three-dimensional space with affine connection $L^{(e)}_3$ with torsion tensor $S^{(e)}{}_{ij}{}^{k}\mathbf{\ni}^i\mathbf{\ni}^j\mathbf{\ni}_k$.

If on the other hand we define a moving reference system and space metric in correspondence with a deformable medium by

$$\mathbf{\ni}^{(p)}{}_i = \frac{\partial \mathbf{r}}{\partial \chi^s}\frac{\partial \chi^s}{\partial \xi^i} \tag{2.12}$$

$$g^{(p)\wedge}{}_{ij} = \mathbf{\ni}^{(p)}{}_i\mathbf{\ni}^{(p)}{}_j, \qquad g^{(p)\wedge}{}_{ij}\,\mathbf{\ni}^{(p)i}\mathbf{\ni}^{(p)j} = g^{(p)}{}_{ij}\,\mathbf{\ni}^i\mathbf{\ni}^j \tag{2.13}$$

Then we obtain formulas similar to (2.10) and (2.11)

$$2S^{(p)}{}_{ij}{}^{k}\mathbf{\ni}_k = (\Gamma^{(p)}{}_{ij}{}^{k} - \Gamma^{(p)}{}_{ji}{}^{k})\,\mathbf{\ni}_k = \frac{D}{Dx^i}\frac{\partial \mathbf{u}}{\partial x^j} - \frac{D}{Dx^j}\frac{\partial \mathbf{u}}{\partial x^i} \tag{2.14}$$

$$\frac{1}{2}\left(\frac{Dg^{(p)}{}_{is}}{Dx^j} + \frac{Dg^{(p)}{}_{js}}{Dx^i} - \frac{Dg^{(p)}{}_{ij}}{Dx^s}\right) - \\ - S^{(p)}{}_{js}{}^{k}g^{(p)}{}_{ki} - S^{(p)}{}_{is}{}^{k}g^{(p)}{}_{kj} + S^{(p)}{}_{ij}{}^{k}g^{(p)}{}_{ks} = \Gamma^{(p)}{}_{ij}{}^{k}g^{(p)}{}_{ks} \tag{2.15}$$

and the space is again found to be a metric space with affine connection $L^{(p)}_3$ and with torsion tensor $S^{(p)}{}_{ij}{}^{k}\mathbf{\ni}^i\mathbf{\ni}^j\mathbf{\ni}_k$. By comparing (2.10) with (2.14), we find

$$S^{(e)}{}_{ij}{}^{k} = -\,S^{(p)}{}_{ij}{}^{k} \tag{2.16}$$

Thus, the continuously dislocated medium under study may be considered, from the geometric point of view, as a space with affine connection $L^{(e)}_3$ or $L^{(p)}_3$, and correspondingly, from a physical point of view, as either elastic with an incompatible strain tensor or inelastic also with an incompatible strain tensor. If the elastic and inelastic strains are considered simultaneously, then the total strains are compatible, and the continuously dislocated medium is, from a geometric point of view, a Euclidean space.

It is easily shown that the curvature tensors of the $L^{(e)}_3$ and $L^{(p)}_3$ spaces equal zero.

All considerations of this section apply to infinitesimal as well as finite strains. However, for finite strains, the formulas (2.8) do not determine the components of elastic deformation, for clearly, in the case of finite strains, such a tensor cannot, in general, be separated from the strain tensor; instead, they determine the components of what may be called a quasi-elastic tensor.

Similarly, in the case of finite strains, the formulas

$$g^{(p)\wedge}{}_{ij} - g^{\circ}{}_{ij} = 2\varepsilon^{(p)\wedge}{}_{ij} \tag{2.17}$$

determine the components of a quasi-parametric tensor.

The torsion tensor $S^{(e)}{}_{ij}{}^{k}\mathbf{\ni}^i\mathbf{\ni}^j\mathbf{\ni}_k$ of the $L^{(e)}_3$ space is, in terms of the theory of continuous dislocations, the tensor of the density of dislocations $\alpha^{ij}\mathbf{\ni}_i\mathbf{\ni}_j$. In fact, from the definition of the dislocation density tensor [1],

$$\mathbf{b} = \oint_C \frac{\partial \mathbf{u}}{\partial x^i}\,dx^i = \int_S \alpha^{ik}\,dS_i\,\mathbf{\ni}_k \tag{2.18}$$

where S is the surface supported on C. By Stokes' theorem, (2.18) yields

$$\alpha^{ik}\mathbf{\ni}_k = e^{ilm}\frac{D}{Dx^l}\frac{\partial \mathbf{u}}{\partial x^m} \tag{2.19}$$

Whence, (2.19) and (2.10) yield

$$\alpha^{ik} = e^{ilm*}S^{(e)}{}_{lm}{}^{k}, \qquad S^{(e)}{}_{ij}{}^{k} = \frac{1}{2}\, e_{lij}\alpha^{lk} \tag{2.20}$$

Here $e^{ilm}\mathfrak{Э}_i\mathfrak{Э}_l\mathfrak{Э}_m$ and $e_{ilm}\mathfrak{Э}^i\mathfrak{Э}^l\mathfrak{Э}^m$ are antisymmetric unit tensors of third order, equal to (+1) for even permutations of the indices, and to (−1) for odd permutations. Formulas similar to (2.20) were previously obtained in [5].

The formulas expressing the components of the torsion tensor in terms of displacements are given in Cartesian coordinates by

$$S^{(e)}{}_{ij}{}^{k} = S^{(p)}{}_{ji}{}^{k} = \frac{1}{2}\, e_{lij}\, e^{lnm} \frac{D}{Dx^n}\, \frac{\partial u^k}{\partial x^m} \tag{2.21}$$

In concluding this section, it should be noted that a continuously dislocated medium may be considered, from the geometric point of view, as a space with affine connection different from $L^{(e)}{}_3$ and $L^{(p)}{}_3$.

In fact, we can introduce a space metric and torsion

$$g^{(1,\ldots,q)\wedge}{}_{ij} = \mathfrak{Э}^{(1,\ldots,q)}{}_i\mathfrak{Э}^{(1,\ldots,q)}{}_j \tag{2.22}$$

$$\mathfrak{Э}^{(1,\ldots,q)}{}_i = \frac{\partial \mathbf{r}}{\partial \chi^s}\Bigg|_{\substack{\chi^m=\text{const}\\ m=1,\ldots,q}} \frac{\partial \chi^s}{\partial \xi^i} \qquad (s = 1, \ldots, (S-q)) \tag{2.23}$$

$$2S^{(1,\ldots,q)}{}_{ij}{}^{k} = \Gamma^{(1,\ldots,q)}{}_{ij}{}^{k} - \Gamma^{(1,\ldots q)}{}_{ji}{}^{k} \tag{2.24}$$

The coefficients of connection are given by

$$\frac{D\mathfrak{Э}^{(1,\ldots,q)}{}_i}{D\xi^j} = \Gamma^{(1,\ldots q)\wedge}{}_{ij}{}^{k}\mathfrak{Э}^{(1,\ldots,q)}{}_k \tag{2.25}$$

In passing to another coordinate system, the coefficients $\Gamma^{(1,\ldots q)\wedge}{}_{ij}{}^{k}$ transform according to transformation formulas of the Christoffel symbols, and therefore they are coefficients of connection. The proof of the above is similar to that given in [3].

The torsion tensor $S^{(1,\ldots q)}{}_{ij}{}^{k}\mathfrak{Э}^i\mathfrak{Э}^j\mathfrak{Э}_k$ is no longer related to the dislocation density tensor by (2.20), since the dislocation density tensor, by definition, characterizes complete incompatibility, when all, and not only certain ones, of the parameters χ^s change simultaneously.

It is readily seen that the number of spaces with affine connection $L^{(1,\ldots q)}{}_3$, including $L^{(p)}{}_3$, equals

$$C_s^1 + C_s^2 + \ldots + C_s^s = 2^s \tag{2.26}$$

where $C_s{}^k$ is the binomial coefficient.

3. Equations of Equilibrium. We repeat here almost verbatim the discussion of Kunin [6]. With the aid of geometric identities interrelating the tensors $g^{(e)}{}_{ij}\mathfrak{Э}^i\mathfrak{Э}^j$, $S^{(e)}{}_{ij}{}^{k}\mathfrak{Э}^i\mathfrak{Э}^j\mathfrak{Э}_k$ and $R^{(e)}{}_{ijk}{}^{l}\mathfrak{Э}^i\mathfrak{Э}^j\mathfrak{Э}^k\mathfrak{Э}_l$ it is possible to obtain the complete set of static equations for the theory of continuous dislocations [2], [4] and [6]

$$\operatorname{Curl} w^{(e)i}{}_j\mathfrak{Э}_i\mathfrak{Э}^j = \alpha^{ij}\mathfrak{Э}_i\mathfrak{Э}_j, \qquad \operatorname{Div} p^{(e)ij}\,\mathfrak{Э}_i\mathfrak{Э}_j = 0, \qquad p^{(e)ij} = \lambda^{(e)ijlm}\, w^{(e)}{}_{lm} \tag{3.1}$$

Here $w^{(e)i}{}_j$ are the components of the elastic distortion tensor

$$w^{(e)i}{}_j = \frac{\partial u^i}{\partial x^j} \tag{3.2}$$

The operations of Curl and Div on a tensor $a^{ij}\ni_i\ni_j$ are defined by

$$\text{Curl}\, a^i{}_j \ni_i \ni^j = e^{ilm} \frac{Da^k{}_m}{Dx^l} \ni_i \ni_k$$

$$\text{Div}\, a^{ij} \ni_i \ni_j = \frac{Da^{ij}}{Dx^i} \ni_j \tag{3.3}$$

The known tensor of the density of dislocations $\alpha^{ij}\ni_i\ni_j$ should be subjected to the condition

$$\text{Div} \alpha^{ij} \ni_i \ni_j = 0 \tag{3.4}$$

Clearly, however, we may obtain similar results with the aid of geometric identities interrelating the tensors

$$g^{(p)}{}_{ij}\, \ni^i \ni^j, \quad S^{(p)}{}_{ij}{}^k \ni^i \ni^j \ni_k, \qquad R^{(p)}{}_{ijk}{}^l \ni^i \ni^j \ni^k \ni_l$$

$$\text{Curl}\, w^{(p)i}{}_j \ni_i \ni^j = -\, \alpha^{ij} \ni_i \ni_j, \quad \text{Div}\, p^{(p)ij} \ni_i \ni_j = 0, \quad p^{(p)ij} = \lambda^{(p)ijlm}\, w^{(p)}{}_{lm} \tag{3.5}$$

Here

$$w^{(p)i}{}_j = \frac{\partial u^i}{\partial \chi^s} \frac{\partial \chi^s}{\partial x^j} \tag{3.6}$$

are the components of the parametric distortion tensor.

If both systems of equations are combined, combining corresponding equations, which is permissible since the equations are linear, we obtain

$$\begin{gathered} \text{Curl}\, w^i{}_j \ni_i \ni^j \equiv 0, \qquad w^i{}_j = w^{(e)i}{}_j + w^{(p)i}{}_j \\ \text{Div}\, p^{ij} \ni_i \ni_j = 0, \qquad p^{ij} = p^{(e)ij} + p^{(p)ij} \\ p^{ij} = \lambda^{(e)ijlm}\, w^{(e)}{}_{lm} + \lambda^{(p)ijlm}\, w^{(p)}{}_{lm} \end{gathered} \tag{3.7}$$

The first equation in (3.7) is satisfied identically; the second and third equations in (3.7) are the equations of equilibrium and equations of state, respectively.

If the medium is in a state of static equilibrium, then the parameters of the medium must satisfy all three systems of equations simultaneously, from which it follows that in a medium with dislocations there must exist not only the elastic stress tensor $p^{(p)ij}\ni_i\ni_j$, defined by (3.2), but also the parametric stress tensor $p^{(p)ij}\ni_i\ni_j$, defined by (3.5).

The system of equations (3.7) is not complete. A complete set of equations for a dynamic system will be obtained in future work.

In conclusion, the author expresses thanks to L.D. Sedov and V.V. Lokhin for their interest in this work and for their valuable advice.

BIBLIOGRAPHY

1. Eshelby J., Kontinual'naia teoriia dislokatsii (Theory of continuous dislocations). Izd-vo inostr. liter., M., 1963.

2. Landau L.D. and Lifshits E.M., Teoriia uprugosti (Theory of elasticity). Izd-vo Nauka, M., 1965.

3. Sedov L.D., Vvedenie v mekhaniku sploshnoi sredi (Introduction to the mechanics of solids). Fizmatgiz, M., 1962.
4. Kosevich A.M., Dinamicheskaia teoria dislokatsii (Dynamic theory of dislocations). UFN, Vol. 84, No. 4, 1964.
5. Kondo K., Memoirs of unifying study of the basic problems in engineering by means of geometry, I, II. Tokyo, Cakujutsu Bunken Fukyn-Kai, 1955, 1958.
6. Kunin I.A., Teoriia dislokatsii (Theory of dislocations). Dopolnenie k monografii (Supplement to the monograph) by Skhouten, J.A., Tenzornyi analiz dlia fizikov (Tensor analysis for physicists). Izd-vo Nauka, M., 1965.

Translated by H.H.